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LETTER TO THE EDITOR

Intensity fluctuation linewidth and the Jakeman–Pike approximation

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Abstract. The Jakeman–Pike approximation for the linewidth of intensity fluctuations is examined in black-body and laser models, and it is shown that this gives a good estimate for use in the design of experiments.

The possibility of observing effects such as bunching and antibunching by photon counting techniques depends on the intensity fluctuation linewidth of the observed radiation. Jakeman and Pike (1971) have proposed a simple prescription (here called the JP approximation) for an effective linewidth which is convenient for practical use because it can be found analytically from the rate equation for the photon density matrix of the system. To judge its validity the JP approximation is examined here firstly in a model for a black-body system, which can be solved exactly; and secondly in the laser system, for which computed data are available (Smith 1975). The JP approximation is found to be exact for the black-body model, and at least a good experimental guide in the laser case.

The JP approximation takes the first term in the Taylor expansion of the logarithm of the intensity autocorrelation function $G^{(2)}(t)$, giving in the steady state

$$G^{(2)}(t) = \langle n\rangle^2 + [\langle n(n-1)\rangle - \langle n\rangle^2] \exp(-\lambda_e t) \tag{1}$$

$$\lambda_e = -[\langle n(n-1)\rangle - \langle n\rangle^2]^{-1} \frac{\mathrm{d}G^{(2)}(t)}{\mathrm{d}t}\bigg|_0 \tag{2}$$

which is exact for $t \to 0$ and $t = \infty$. $\langle\ \ \rangle$ denote steady-state averages. An exact form is

$$G^{(2)}(t) = \sum_{m,n} mnP_m(\infty)P_n(m-1|t) \tag{3}$$

where $P_m(\infty)$ is the equilibrium photon distribution and $P_n(m-1|t)$ is the conditional probability of finding an $|n\rangle$ photon state at a time t after finding the photon state $|m\rangle$. It must be noted that the observation of a photon in state $|m\rangle$ reduces this to $|m-1\rangle$. The probability $P_n(m|t)$ satisfies the same rate equation as $P_n(t)$ but with initial condition $P_n(m|0) = \delta_{mn}$.

In the back-body case it is more convenient to use the one-photon and conditional one-photon densities defined by the first moments of the photon distributions:

$$\bar{n}(t) = \sum_n nP_n(t), \qquad \bar{n}(m|t) = \sum_n nP_n(m|t). \tag{4}$$

The usual single-mode black-body rate equation (Loudon 1973) leads to the rate equation

$$\frac{\mathrm{d}\bar{n}(t)}{\mathrm{d}t} = -\lambda_s \bar{n}(t) + A, \qquad \lambda_s = C - A. \tag{5}$$

A and C respectively are the constants which determine the rates at which photon states are raised and lowered. This equation can be modified to give a simple model of an absorption experiment. If there is an increase of L in the rate at which photons are emitted, and if radiation enters the simple system at a rate R in a specified steady state labelled by I, then considering the flow of photons as a continuity problem with the right-hand side of equation (5) as a source term:

$$\frac{\mathrm{d}\bar{n}(t)}{\mathrm{d}t} = -\lambda \bar{n}(t) + A + R\langle n\rangle^{\mathrm{I}}, \qquad \lambda = \lambda_s + L. \tag{6}$$

Noting that $\bar{n}(\infty) = \langle n\rangle$ the solution of equation (6) is

$$\bar{n}(t) = \langle n\rangle + (n(0) - \langle n\rangle)\exp(-\lambda t). \tag{7}$$

When the intensity fluctuation linewidth of the incident radiation is very much greater or very much less than λ then the conditional one-photon density $\bar{n}(m|t)$ is also given by equation (6) (the general case is under consideration by the authors). The initial condition for the conditional density is given by that for the conditional probability as $\bar{n}(m|0) = m$. Substituting in equation (3):

$$G^{(2)}(t) = \langle n\rangle^2 + [\langle n(n-1)\rangle - \langle n\rangle^2]\exp(-\lambda t) \tag{8}$$

and $\lambda = C - A + L$ is therefore the exact linewidth. Using equation (6) in the JP approximation (2) it is also found that

$$\lambda_e = C - A + L = \lambda. \tag{9}$$

Hence not only is the JP approximation for the linewidth λ_e correct, but the analytic form (1) is also exact in this model. Although the above results are given for the steady state only minor modifications are needed for non-steady states and the identity of linewidths and analytic forms remains true.

Analytic solution of the simple black-body rate equation (with $L = R = 0$) using the generating function method (see Agarwal 1970) gives an infinite series of decay modes with $\lambda_i = i(C - A)$ (i positive integer) and computer calculations (G P Hildred 1977, unpublished) indicates that many of these (~ 20) contribute to the decay of the conditional probability. The decay of $G^{(2)}(t)$ is therefore considerably simpler than the decay of the conditional probability, and this enables the JP approximation to be effective.

For the Scully–Lamb laser rate equation (Scully *et al* 1966, Scully and Lamb 1967) the JP approximation for λ_e gives (Jakeman and Pike 1971)

$$\frac{\lambda_e}{\lambda_0} = \frac{\langle n\rangle(\langle n^2\rangle_0 - \langle n\rangle_0^2)}{\langle n\rangle_0(\langle n^2\rangle - \langle n\rangle^2)} \tag{10}$$

λ_0, $\langle n\rangle_0$, $\langle n^2\rangle_0$ being threshold values. The computed data of Smith (1975) enable the right- and left-hand sides of equation (10) to be evaluated independently. The results are given in table 1 for three saturation parameters S, and intensities below and above threshold.

Table 1.

$\langle n\rangle/\langle n\rangle_0$	$S=\infty$		$S=1600$		$S=100$	
	LHS	RHS	LHS	RHS	LHS	RHS
0·1	6·2	5·9	5·0	6·0	3·1	6·1
0·4	1·8	1·7	1·8	1·7	1·6	1·8
0·8	1·1	1·1	1·1	1·1	1·1	1·1
2·0	0·9	1·0	0·9	1·0	0·8	0·9
4·0	1·4	1·4	1·3	1·4	1·0	1·0
10·0	3·8	2·3	3·0	2·3	2·0	2·3

These results show that the JP approximation is at worst a good experimental guide to the linewidth likely to be found in the system. The largest disagreement occurs well below threshold for the system with low threshold; but the applicability of the Scully–Lamb equation in this case is in doubt (Smith 1975).

It may be concluded that where the intensity fluctuation linewidth is needed for the design of experiments, such as bunching or antibunching by photon counting techniques, then the JP approximation provides a good estimate.

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